A Recommended Simple and General Formula for the Specification of the Efficiency of Regulated dc-to-dc Converters

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1. Introduction

A very simple circuit model is presented that captures the dominant loss mechanisms in regulated, PWM, dc-to-dc converters. Using this circuit, a simple efficiency formula is then derived which fits remarkably well the efficiency data of single output and multiple output, regulated dc-to-dc converters. Representative COTS converters from **Vicor**, **Delphi** and **Murata** are used to verify the efficiency formula for single output converters. Efficiency measurements on custom designed high power, high voltage and multiple output converters are presented and shown to agree very well with the simple formula presented in this work.

The proposed efficiency formula has two loss parameters, P_{sc} and P_{oh}, which are *measured* on the converter and *not* computed from the schematic design of the converter. P_{oh} is measured at no load while P_{sc} is measured at maximum load. When these parameters are substituted in the efficiency formula, the correct efficiency is obtained at any output power. When the formula is generalized to multiple output converters, a P_{sc} is determined for each output while P_{oh} remains the same as before as will be explained.

It is recommended that these two parameters be specified in the data sheet of COTS dc-to-dc converters to fully characterize their efficiency at any load. Manufacturers of dc-to-dc converters typically specify the efficiency at maximum load and occasionally provide efficiency curves for single output converters. For multiple output converters, efficiency curves cannot be provided as a function of the output power simply because the total output power can be any combination of the individual output powers. The efficiency formula for multiple outputs given here allows one to compute the efficiency at any output power for any combination of the output powers.

2. Loss model and efficiency of a regulated dc-to-dc converter

The simple circuit model with two loss elements shown in **Figure 1** is proposed for the determination of the efficiency characteristics of a regulated dc-to-dc converter. The current source I_{oh} is a dc current source such that $V_{in}I_{oh}$ represents the overhead or no load ($I_{out} = 0$) losses of the converter and is determined by a measurement of the input power to the converter at no load and at a certain input voltage:

$$I_{oh} = \frac{P_{oh}}{V_{in}} \tag{1}$$

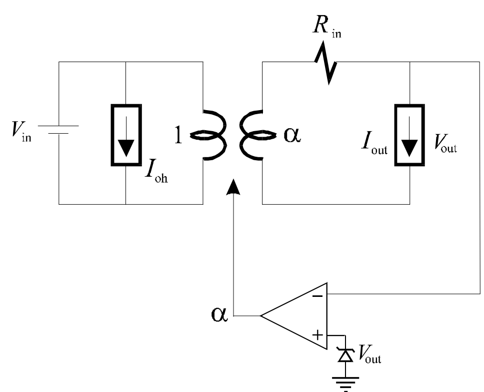


Figure 1

An approximate justification for the use of a current source for the overhead losses is that the power consumed by most switching converters, regardless of their topology, under no load conditions is in the control and gate-drive circuits which appear essentially as current sources to the house keeping supply which powers them.

The term R_{int} represents all other losses in the converter associated with the output load current and is determined from the experimental data of the converter at the maximum output power for a given input voltage according to:

$$R_{int} = \frac{V_{in}I_{in} - V_{out}I_{o\max} - P_{oh}}{I_{o\max}^{2}} \tag{2}$$

An approximate justification for the use of R_{int} for load-current dependent losses is that the dominant component of these losses, in any switching converter, essentially consists of copper and conduction losses which are proportional to the *square* of the current rather than linearly proportional to the current as may be the case in the switching losses. The combination of these two loss parameters appears to account fairly well for the loss characteristics of most switching regulators as will be shown.

The efficiency of the circuit shown in **Figure 1** is now determined as follows. The output power, the losses and the input power are given by:

$$P_{out} = V_{out} I_{out}$$
$$P_{loss} = I_{out}^2 R_{\text{int}} + V_{in} I_{oh}$$
$$P_{in} = V_{out} I_{out} + I_{out}^2 R_{\text{int}} + V_{in} I_{oh}$$

The efficiency is given by:

$$\begin{aligned} \eta(P_{out}) &= \frac{P_{out}}{P_{out} + I_{out}^2 R_{\text{int}} + V_{in} I_{oh}} \\ &= \frac{1}{1 + \frac{I_{out}^2 R_{\text{int}}}{P_{out}} + \frac{V_{in} I_{oh}}{P_{out}}} \\ &= \frac{1}{1 + \frac{I_{out}^2 R_{\text{int}}}{V_{out} I_{out}} \frac{V_{out}}{V_{out}} + \frac{V_{in} I_{oh}}{P_{out}}} \end{aligned} \tag{3}$$

Since the output voltage is constant, it is convenient to introduce the following definition:

$$P_{sc} \equiv \frac{V_{out}^2}{R_{int}} = \frac{P_{o\_max}^2}{P_{in\_max} - P_{o\_max} - P_{oh}} \tag{4}$$

The efficiency is now given by:

$$\boxed{\eta(P_{out}) = \frac{1}{1 + \frac{P_{oh}}{P_{out}} + \frac{P_{out}}{P_{sc}}}} \tag{5}$$

It is clear that Eq. (5) is not derived from first principles, but one which is derived from a loss circuit model proposed by heuristic arguments. It is an equation which can be easily determined from two data points for each input voltage. Hence to obtain a family of efficiency curves at each input voltage of a particular converter using Eq. (5), we need two *data points*, P_{sc} and P_{oh}, for each input voltage.

Finally, it can be shown that the maximum efficiency in Eq. (5) and the output power at which it occurs are given by:

$$\eta_{\max} = \frac{1}{1 + 2\sqrt{\dfrac{P_{oh}}{P_{sc}}}} \tag{6}$$

$$P_{\eta\_\max} = \sqrt{P_{sc} P_{oh}} \tag{7}$$

Often this maximum is quite shallow and sometimes may occur outside the maximum rating of the converter, which suggests that the converter has been over-designed for that maximum power.

For multiple output converters, the efficiency formula in Eq. (5) can be generalized as follows:

$$\boxed{\eta(P_o) = \frac{1}{1 + \dfrac{1}{P_o}\left(P_{oh} + \dfrac{P_{o\_1}^2}{P_{sc\_1}} + \dfrac{P_{o\_2}^2}{P_{sc\_2}} + \dfrac{P_{o\_3}^2}{P_{sc\_3}} + \cdots\right)}} \tag{8}$$

in which:

$$P_o \equiv \text{Sum of all output powers} = P_{o\_1} + P_{o\_2} + P_{o\_3} + \cdots$$

$$P_{oh} \equiv \text{Input power at no load on all outputs: } P_{o\_i} = 0$$

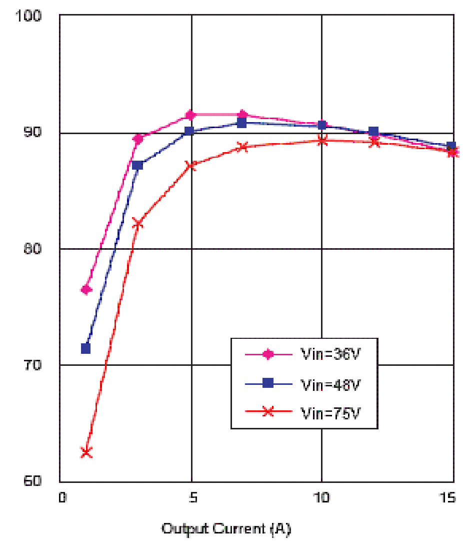
$$P_{sc\_j} \equiv \frac{P_{o\_j\_max}^2}{P_{in\_max} - P_{o\_j\_max} - P_{oh}} \quad ; \quad \begin{cases} P_{o\_i} = 0 \\ i \neq j \end{cases} \tag{9}$$

In the case of multiple output converters, an efficiency curve as a function of the total output power cannot be given because the total output power can be any combination of the individual output powers, but Eq. (8) can fully characterize the efficiency of a multiple output converter for any output load in any combination of the individual output powers.

3. Modeling the efficiency characteristics of COTS dc to dc converters

a) <u>MURATA Model MPDKN007S, 3.3V, 50W dc-to-dc converter</u>

The efficiency data in Table **I** of this converter is extracted from the efficiency graph provided by the manufacturer [2] shown here in **Figure 2a**.



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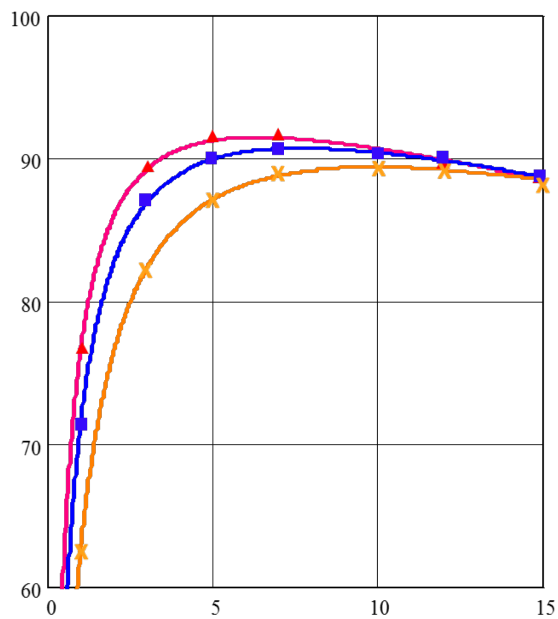


(*b*)

Figure 2 Efficiency curves of Model MPDKN007S *a*) measured data as provided by MURATA *b*) The efficiency curve given by Eq. (5) with the measured values shown in **Figure 2a** transcribed and superimposed for comparison.

Table I Efficiency of Model MPDKN007S at $27^{\circ}C$

I_{out}	V_{in} = 36V	V_{in} = 48V	V_{in} = 75V
1A	76.6%	71.4%	62.6%
3A	89.4%	87.2%	82.0%
5A	91.4%	90.1%	86.9%
7A	91.4%	90.7%	88.0%
10A	90.6%	90.5%	90.0%
12A	90.0%	90.0%	89.0%
15A	88.5%	88.7%	88.5%

The values of I_{oh} are determined to be:

$$\begin{aligned} I_{oh} &= 27.3\text{mA} \;\; ; \;\; V_{in} = 36\text{V} \\ I_{oh} &= 27.0\text{mA} \;\; ; \;\; V_{in} = 48\text{V} \\ I_{oh} &= 26.0\text{mA} \;\; ; \;\; V_{in} = 75\text{V} \end{aligned} \tag{10}$$

The calculated values of R_{int} using Eq. (2) are as follows:

$$\begin{aligned} R_{int} &= 24m\Omega \;\; ; \;\; V_{in} = 36V \\ R_{int} &= 22m\Omega \;\; ; \;\; V_{in} = 48V \\ R_{int} &= 20m\Omega \;\; ; \;\; V_{in} = 75V \end{aligned} \tag{11}$$

The resultant plots of the efficiency curves using Eq. (5) are shown in **Figure 2b** and the difference between Eq. (5) and the measured efficiency are shown in **Table II**.

Notice again that the parameters I_{oh} and R_{int} are not strong functions of the input voltage so that one can use either a single average value or an interpolated value as a function of the input voltage.

Table II Difference between measured efficiency and Eq. (5) for Model MPDKN007S

I_{out}	V_{in} = 36V	V_{in} = 48V	V_{in} = 75V
1A	0.02%	0.06%	0.02%
3A	-0.22%	-0.33%	0.30%
5A	-0.18%	-0.20%	0.18%
7A	0.02%	-0.07%	0.76%
10A	0.05%	-0.15%	-0.67%
12A	-0.14%	-0.21%	0.15%
15A	0%	0%	0%

Hence, the efficiency of this converter can be specified in its data sheet by providing the following table:

Table III

MPDKN007S	$V_{in} = 36V$	$V_{in} = 48V$	$V_{in} = 75V$
P_{oh}	0.9828W	1.296W	1.95W
P_{sc}	453.75W	495W	544.5W

b) MURATA Model MPDKN004S, 1.8V, 30W dc-to-dc converter

The efficiency data of this converter given in **Table IV** is extracted from the efficiency graph provided by the manufacturer [2] and shown here in **Figure 3a**.

Table IV Efficiency of Model MPDKN004S at $27^{\circ}C$

I_{out}	V_{in} = 36V	V_{in} = 48V	V_{in} = 75V
2.91A	83.3%	79.3%	73.2%
5.00A	87.1%	84.8%	80.2%
6.94A	88.3%	86.9%	83.1%
10.0A	88.3%	87.3%	85.1%
11.3A	87.8%	87.3%	85.5%
15A	86.7%	86.6%	85.5%

The values of I_{oh} are determined to be:

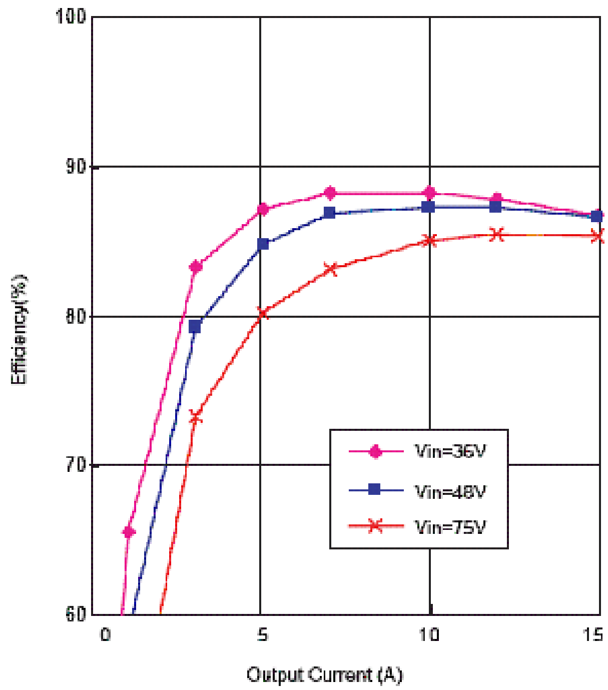
$$\begin{aligned} I_{oh} &= 25.8\text{mA} \quad ; \quad V_{in} = 36\text{V} \\ I_{oh} &= 26.5\text{mA} \quad ; \quad V_{in} = 48\text{V} \\ I_{oh} &= 25.5\text{mA} \quad ; \quad V_{in} = 75\text{V} \end{aligned} \tag{12}$$

The calculated values of R_{int} using Eq. (2) are as follows:

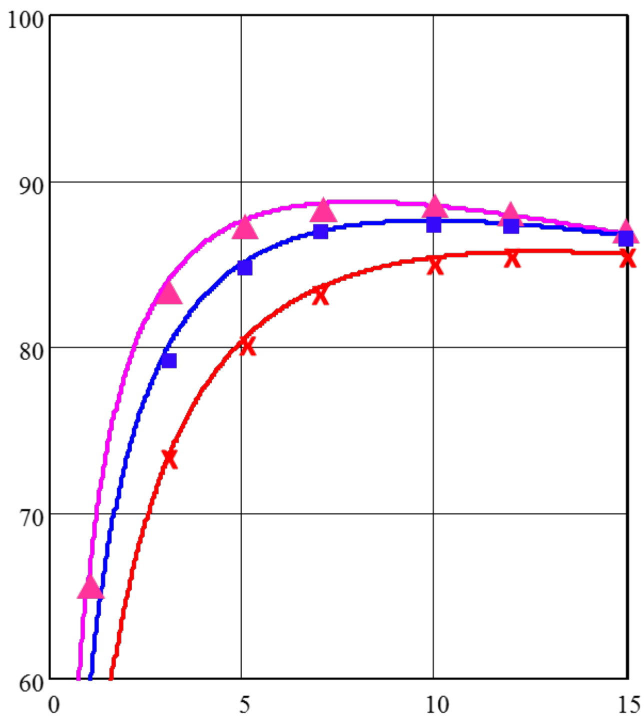
$$\begin{aligned} R_{\text{int}} &= 14\text{m}\Omega \quad ; \quad V_{in} = 36\text{V} \\ R_{\text{int}} &= 13\text{m}\Omega \quad ; \quad V_{in} = 48\text{V} \\ R_{\text{int}} &= 12\text{m}\Omega \quad ; \quad V_{in} = 75\text{V} \end{aligned} \tag{13}$$

The resultant plots of the efficiency curves are shown in **Figure 3b** and the difference between Eq. (5) and the measured efficiency are shown in **Table V**.

Notice again that the parameters I_{oh} and R_{int} are not strong functions of the input voltage so that one can use either a single average value or an interpolated value as a function of the input voltage.



(*a*)



(*b*)

Figure 3 Efficiency curves of Model MPDKN004S measured data as provided by MURATA *b*) The efficiency curve given by Eq. (5) with the measured values shown in **Figure 3a** transcribed and superimposed for comparison.

Table V Difference between measured efficiency and Eq. (5)
For MURATA Model MPDKN004S at $27^{\circ}C$

I_{out}	V_{in} = 36V	V_{in} = 48V	V_{in} = 75V
2.91A	0%	-0.17%	-0.96%
5.00A	0.40%	0.147%	0.094%
6.94A	0.24%	-0.063%	0.323%
10.0A	0.12%	0.234%	0.218%
11.3A	0.16%	0.086%	0.128%
15A	0%	0%	0%

Hence, the efficiency of this converter can be specified in its data sheet by providing the following table.

Table VI

MPDKN004S	$V_{in} = 36V$	$V_{in} = 48V$	$V_{in} = 75V$
P_{oh}	0.9288W	1.272W	1.912W
P_{sc}	231.4W	249.2W	270W

c) MURATA Model MPD6D207S, 3.3V, 30W dc-to-dc converter

The efficiency date of this converter given in **Table VII** is extracted from the efficiency graph in **Figure 4a** provided by the manufacturer [3].

Table VII Efficiency data of Murata Model MPD6D207S

I_{out}	V_{in} = 18V	V_{in} = 24V	V_{in} = 30V	V_{in} = 36V
0.9A	80.0%	80.0%	76.3%	73.3%
.8A	87.7%	87.7%	85.7%	83.8%
2.7A	90.3%	90.4%	88.9%	87.6%
3.6A	91.4%	91.5%	90.3%	89.4%
4.5A	91.8%	92.0%	91.2%	90.3%
5.4A	91.9%	92.2%	91.6%	90.8%
6.3A	91.5%	92.2%	91.7%	91.0%
7.2A	91.1%	92.0%	91.7%	91.1%
8.1A	91.0%	91.6%	91.5%	90.9%
9.0A	90.3%	91.2%	91.0%	90.8%

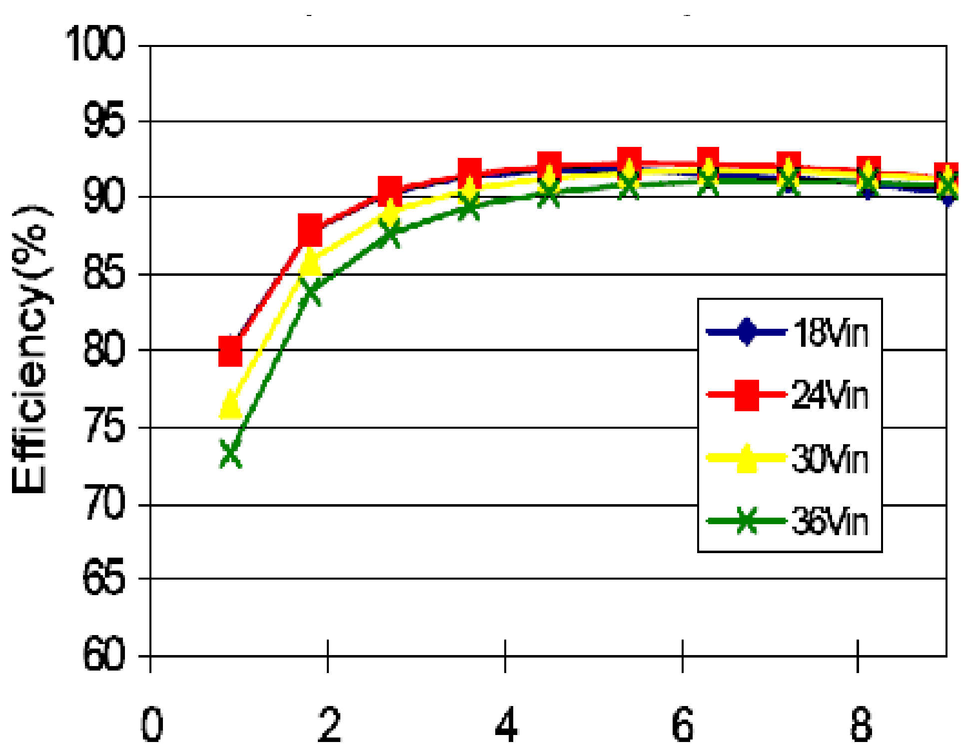
The values of I_{oh} are determined to be:

$$\begin{aligned} I_{oh} &= 40.0\text{mA} \;\; ; \;\; V_{in} = 18\text{V} \\ I_{oh} &= 30.0\text{mA} \;\; ; \;\; V_{in} = 24\text{V} \\ I_{oh} &= 30.0\text{mA} \;\; ; \;\; V_{in} = 30\text{V} \\ I_{oh} &= 29.5\text{mA} \;\; ; \;\; V_{in} = 36\text{V} \end{aligned} \tag{14}$$

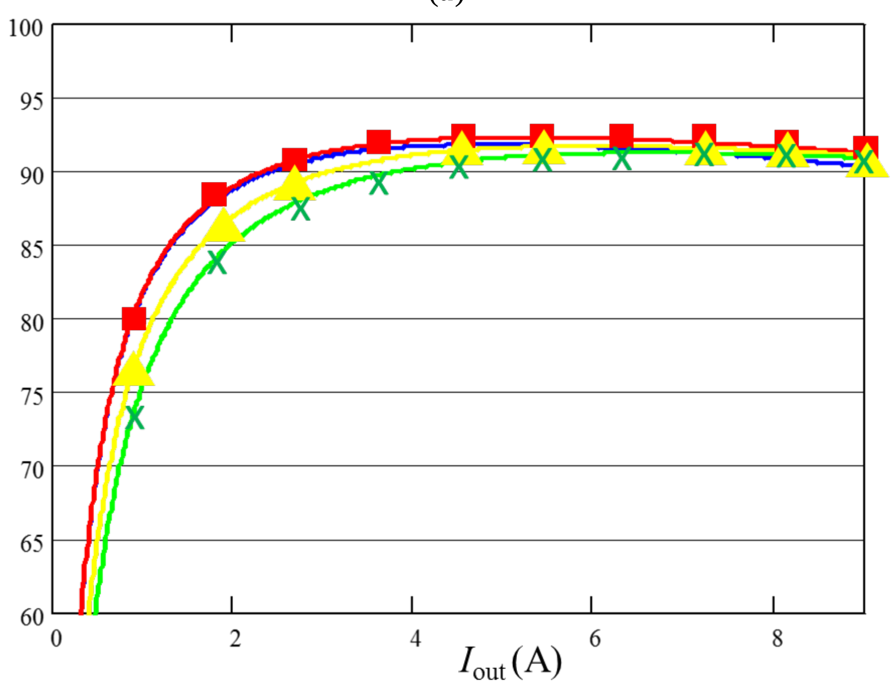
The calculated values of R_{int} using Eq. (2) are as follows:

$$\begin{aligned} R_{\text{int}} &= 30\text{m}\Omega \;\; ; \;\; V_{in} = 18\text{V} \\ R_{\text{int}} &= 26\text{m}\Omega \;\; ; \;\; V_{in} = 24\text{V} \\ R_{\text{int}} &= 25\text{m}\Omega \;\; ; \;\; V_{in} = 30\text{V} \\ R_{\text{int}} &= 24\text{m}\Omega \;\; ; \;\; V_{in} = 36\text{V} \end{aligned} \tag{15}$$

The resultant plots of the efficiency curves are shown in **Figure 4b** and the difference between Eq. (5) and the measured efficiency are shown in **Table VIII.**



(*a*)



(*b*)

Figure 4 *a*) Measured data efficiency curves of Model MPD6D207S as a function of the output current (Scale Amps/div) provided by MURATA and *b*) The efficiency curve given by Eq. (5) with the measured values shown in **Figure 4a** transcribed and superimposed for comparison.

Table VIII Difference between measured efficiency and Eq. (5)

I_{out}	V_{in} = 18V	V_{in} = 24V	V_{in} = 30V	V_{in} = 36V
0.9A	-0.05	0.02	0.04	0.00
1.8A	0.19	0.35	0.12	0.1
2.7A	0.14	0.30	0.26	0.21
3.6A	0.02	0.29	0.35	0.24
4.5A	-0.06	0.2	0.13	0.26
5.4A	0.18	0.07	0.00	0.20
6.3A	0.00	-0.05	-0.07	0.16
7.2A	0.07	-0.10	-0.19	0.05
8.1A	-0.24	-0.02	-0.21	0.11
9.0A	0	0	0	0

Notice again that the parameters I_{oh} and R_{int} are not strong functions of the input voltage so that one can use either a single average value or an interpolated value as a function of the input voltage.

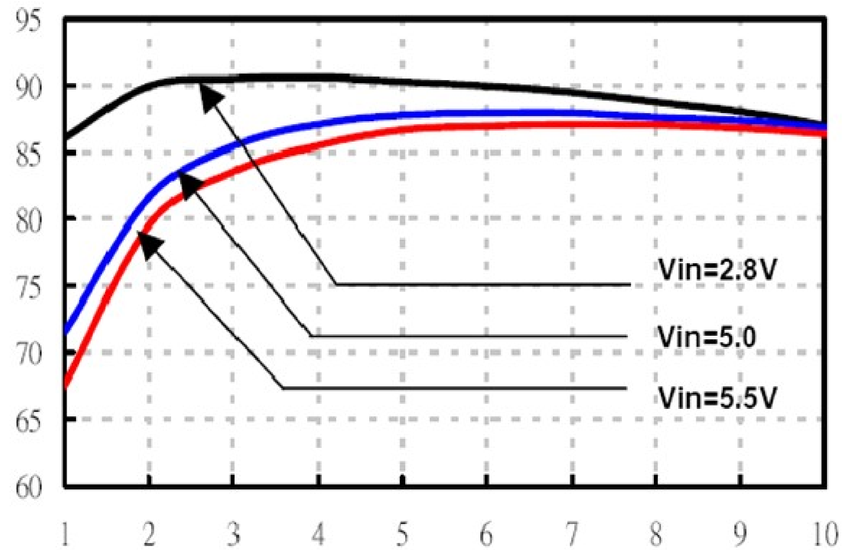
Hence, the efficiency of this converter can be specified in its data sheet by providing the following table:

Table IX

MPD6D207S	$V_{in} = 18V$	$V_{in} = 24V$	$V_{in} = 30V$	$V_{in} = 36V$
P_{oh}	0.72W	0.72W	0.9W	1.06W
P_{sc}	363W	418.8W	435.6W	453.8W

d) DELPHI Model 0.75V, 10A dc-to-dc converter

The efficiency curves of this converter, provided by the manufacturer [4], are shown in **Figure 5a.**



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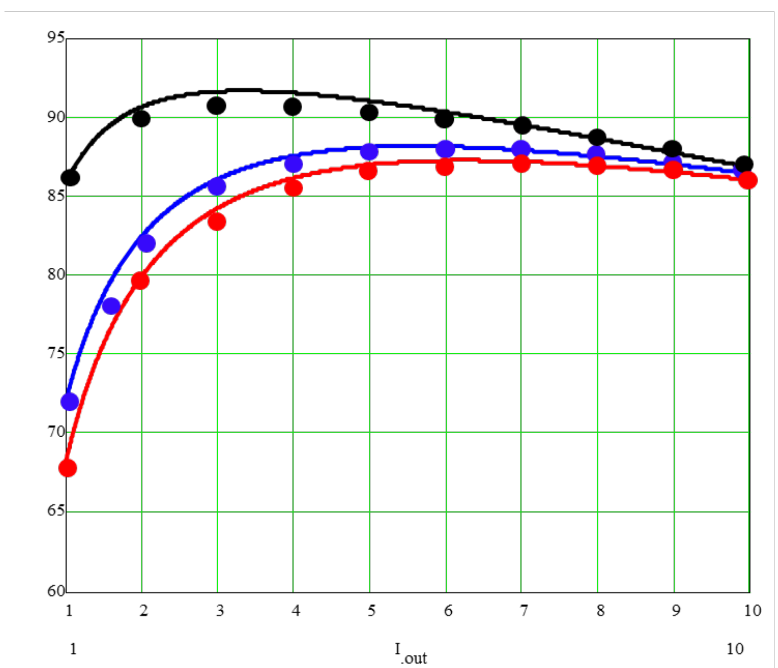


(*b*)

Figure 5 Efficiency curves of Delphi 0.75V Model *a*) measured data as provided by Delta *b*) The efficiency curve given by Eq. (5) with the measured values shown in **Figure 5a** transcribed and superimposed for comparison.

The values of I_{oh} are determined to be:

$$\begin{aligned} I_{oh} &= 41.0\text{mA} \;\; ; \;\; V_{in} = 2.8\text{V} \\ I_{oh} &= 57.0\text{mA} \;\; ; \;\; V_{in} = 5.0\text{V} \\ I_{oh} &= 62.5\text{mA} \;\; ; \;\; V_{in} = 5.5\text{V} \end{aligned} \tag{16}$$

The calculated values of R_{in} using Eq. (2) are as follows:

$$\begin{aligned} R_{\text{int}} &= 10\text{m}\Omega \;\; ; \;\; V_{in} = 2.8\text{V} \\ R_{\text{int}} &= 8.956\text{m}\Omega \;\; ; \;\; V_{in} = 5.0\text{V} \\ R_{\text{int}} &= 8.772\text{m}\Omega \;\; ; \;\; V_{in} = 5.5\text{V} \end{aligned} \tag{17}$$

The resultant plots of the efficiency curves are shown in **Figure 5b.**

Notice again that the parameters I_{oh} and R_{int} are not strong functions of the input voltage so that one can use either a single average value or an interpolated value as a function of the input voltage.

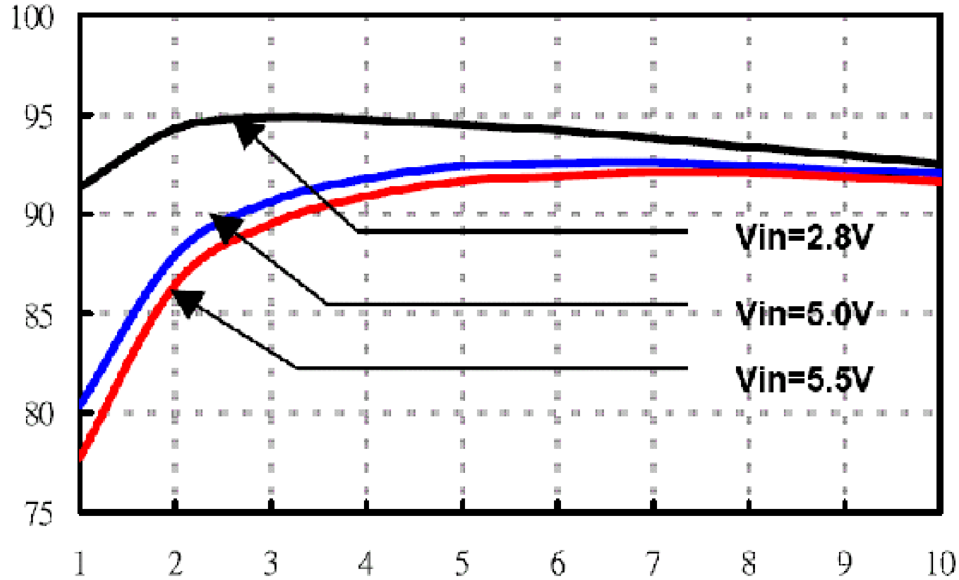
Hence, the efficiency of this converter can be specified in its data sheet by providing the following table:

Table X

Delphi 0.75V	$V_{in} = 2.8V$	$V_{in} = 5V$	$V_{in} = 5.5V$
P_{oh}	0.1176W	0.285W	0.3438W
P_{sc}	56.25W	62.81W	64.98W

e) DELPHI Model 1.5V, 10A dc-to-dc converter

The efficiency curves of this converter, provided by the manufacturer [4], are shown in **Figure 6a** .



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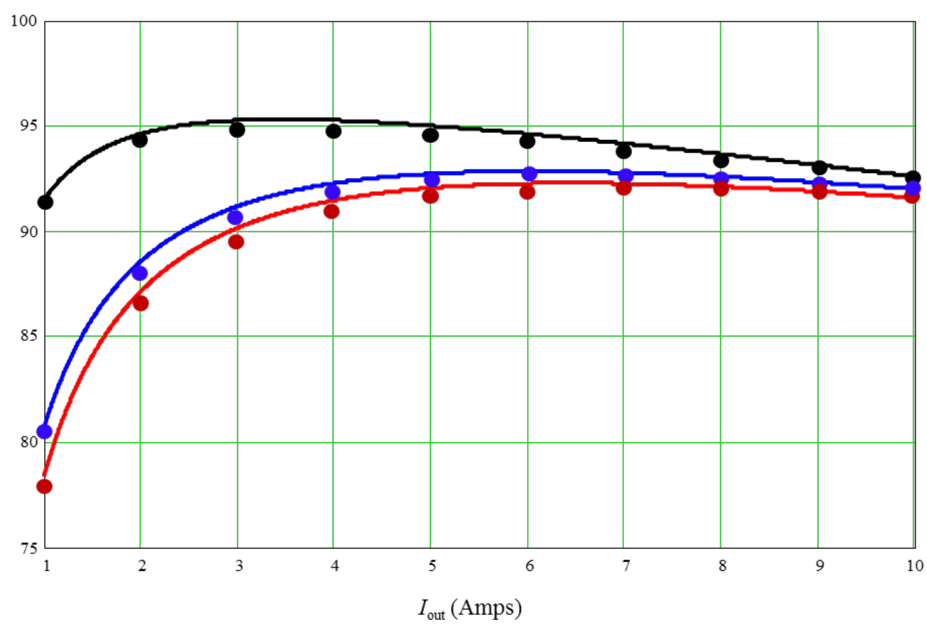


(*b*)

Figure 6 Efficiency curves of Delphi 1.5V Model *a*) measured data as provided by Delta *b*) The efficiency curve given by Eq. (5) with the measured values shown in **Figure 6a** transcribed and superimposed for comparison.

The values of I_{oh} are determined to be:

$$\begin{aligned} I_{oh} &= 46.0\text{mA} \quad ; \quad V_{in} = 2.8\text{V} \\ I_{oh} &= 70.0\text{mA} \quad ; \quad V_{in} = 5.0\text{V} \\ I_{oh} &= 73.5\text{mA} \quad ; \quad V_{in} = 5.5\text{V} \end{aligned} \tag{18}$$

The calculated values of R_{int} using Eq. (2) are as follows:

$$\begin{aligned} R_{\text{int}} &= 11\text{m}\Omega \quad ; \quad V_{in} = 2.8\text{V} \\ R_{\text{int}} &= 9.543\text{m}\Omega \quad ; \quad V_{in} = 5.0\text{V} \\ R_{\text{int}} &= 9.713\text{m}\Omega \quad ; \quad V_{in} = 5.5\text{V} \end{aligned} \tag{19}$$

The resultant plots of the efficiency curves are shown in **Figure 6b** and compared to the measured values.

Notice again that the parameters I_{oh} and R_{int} are not strong functions of the input voltage so that one can use either a single average value or an interpolated value as a function of the input voltage.

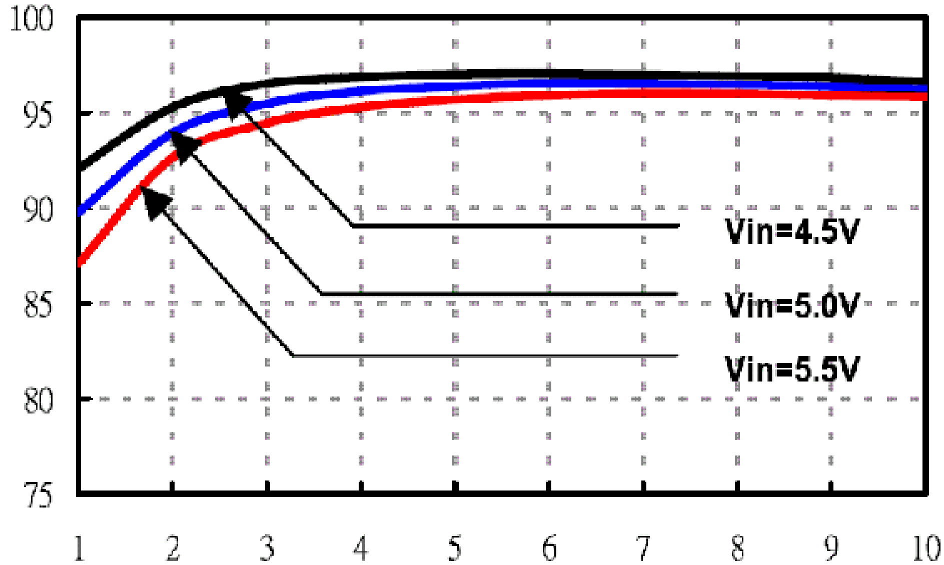
Hence, the efficiency of this converter can be specified in its data sheet by providing the following table:

Table XI

Delphi 1.5V	$V_{in} = 2.8V$	$V_{in} = 5V$	$V_{in} = 5.5V$
P_{oh}	0.1288W	0.35W	0.4043W
P_{sc}	204.5W	235.6W	231.6W

f) DELPHI Model 3.3V, 10A dc-to-dc converter

The efficiency curves of this converter, provided by the manufacturer [4], are shown in **Figure 7a**.



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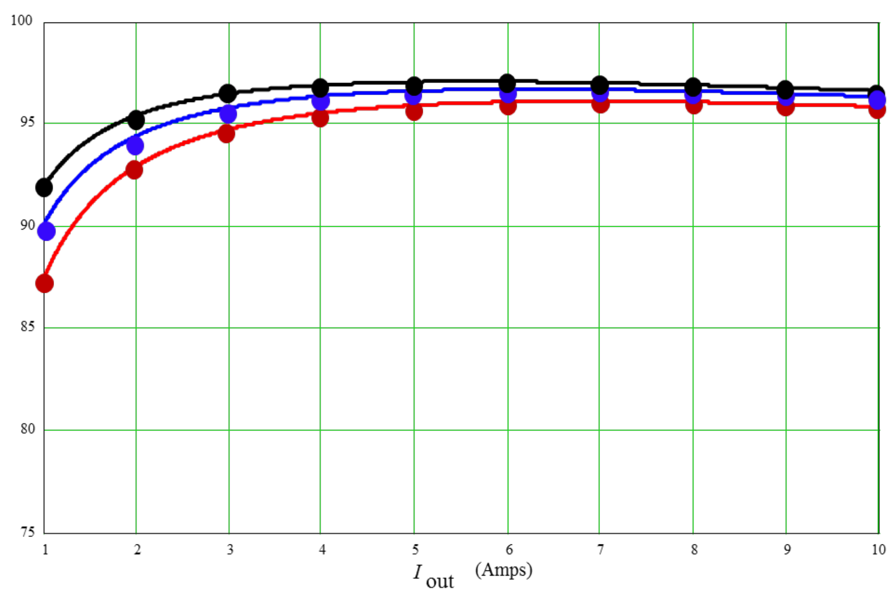

(*b*)

Figure 7 Efficiency curves of Delphi 3.3V Model *a*) measured data as provided by Delta *b*) The efficiency curve given by Eq. (5) with the measured values shown in **Figure 7a** transcribed and superimposed for comparison.

The values of I_{oh} are determined to be:

$$\begin{aligned} I_{oh} &= 63.0\text{mA} \;\;;\;\; V_{in} = 4.5\text{V} \\ I_{oh} &= 71.5\text{mA} \;\;;\;\; V_{in} = 5.0\text{V} \\ I_{oh} &= 85.0\text{mA} \;\;;\;\; V_{in} = 5.5\text{V} \end{aligned} \tag{20}$$

The calculated values of R_{int} using Eq. (2) are as follows:

$$\begin{aligned} R_{\text{int}} &= 8.78\text{m}\Omega \;\;;\;\; V_{in} = 4.5\text{V} \\ R_{\text{int}} &= 9.10\text{m}\Omega \;\;;\;\; V_{in} = 5.0\text{V} \\ R_{\text{int}} &= 9.79\text{m}\Omega \;\;;\;\; V_{in} = 5.5\text{V} \end{aligned} \tag{21}$$

The resultant plots of the efficiency curves are shown in **Figure 7b**.

Notice again that the parameters I_{oh} and R_{int} are not strong functions of the input voltage so that one can use either a single average value or an interpolated value as a function of the input voltage

Hence, the efficiency of this converter can be specified in its data sheet by providing the following table:

Table XII

Delphi 3.3V	$V_{in} = 2.8V$	$V_{in} = 5V$	$V_{in} = 5.5V$
P_{oh}	0.2835W	0.3575W	0.4675W
P_{sc}	1240.3W	1196.7W	1112.3

g) VICOR Model PI3101-00-HVIZ: 3.3V/60W/ 28V input

This is a single output converter with the following input and output voltage and currents:

$V_{in} = 36V$ to $60V$

$V_{out} = 3.3V$

$P_{out} = 0$ to $59.4W$ $(0 - 18\text{Amps})$

The following electrical characteristics are copied from the data sheet of this converter.

PI3101-00-HVIZ Electrical Characteristics

Unless otherwise specified: 36V < V_{IN} < 75V, 0A < I_{OUT} < 18A, –40°C < T_{CASE} < 100°C [a]

Parameter	Symbol	Conditions	Min	Typ	Max	Unit
Input Specifications						
Input Idling Power	P_{IDLE}	V_{IN} = 48V, I_{OUT} = 0A		4		W
Input Standby Power	P_{SBY}	V_{IN} = 48V, ENABLE = 0V		0.120		W
Input Current Full Load	I_{IN}	T_{CASE} = 100°C, I_{OUT} = 18A, η_{FL} = 86.5% typical, V_{IN} = 48V		1.43		A_{DC}

From the idling power shown in the data sheet, we can compute P_{oh}:

$$P_{oh} = 4W$$

From the maximum power specified at 48V input voltage in the data sheet above, we can compute P_{sc} as follows:

$$P_{sc} \equiv \frac{P_{o\_max}^2}{P_{in\_max} - P_{o\_max} - P_{oh}} = \frac{(3.3V \times 18A)^2}{48V \times 1.43A - 3.3V \times 18A - 4W} = 671.3W$$

When we substitute these parameters in the efficiency formula given by Eq. (5), we get:

$$\eta = \frac{1}{1 + \frac{4}{P_{out}} + \frac{P_{out}}{671.3}}$$

This equation is plotted in **Figure 8** in which the agreement with the measured efficiency is seen to be excellent.

Hence, the two parameters can be easily specified in the data sheet for each input voltage.

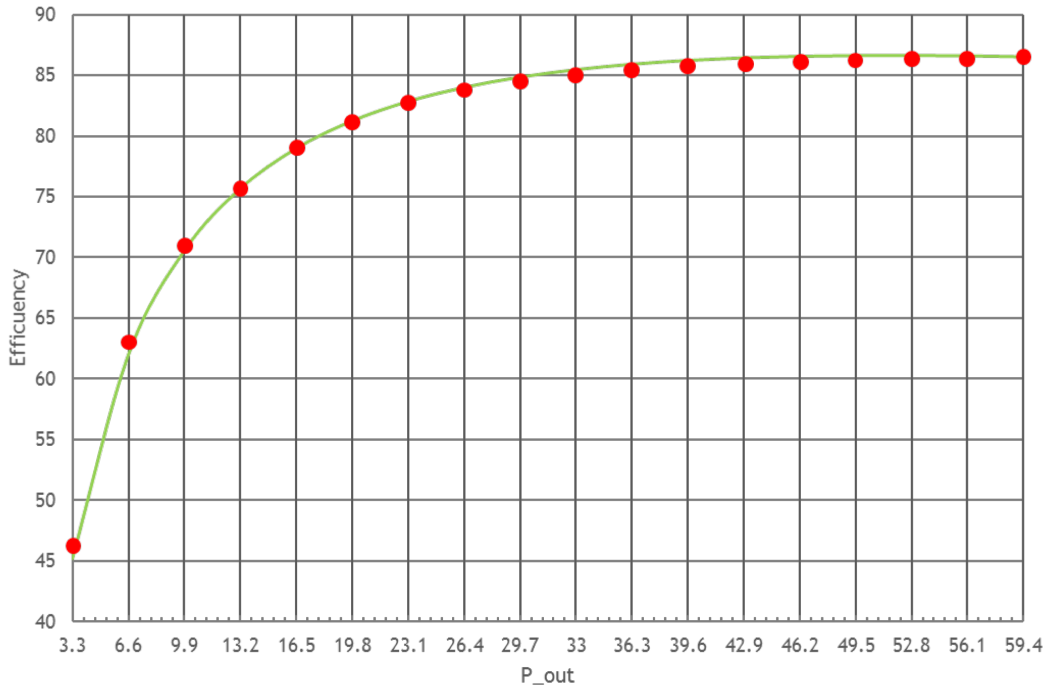

Figure 8 Measured values of the Vicor PI3101-00-HVIZ converter (red dots) and the efficiency curve given by Eq. (5).

4. Experimental results for custom design converters

In this section, efficiency data of two custom design regulated dc-to-dc converters are presented

a) A 3kW, 3kV-to-625V dc-to-dc converter

This converter was built by SAIC for the STAFAC project based on the architecture described in [1] for the Neptune project. The no load input current for three input voltages in the range 2.5kV to 3.5kV was measured as follows:

$$\begin{aligned} I_{oh} &= 47.0\text{mA} \;\; ; \;\; V_{in} = 2.5\text{kV} \\ I_{oh} &= 44.0\text{mA} \;\; ; \;\; V_{in} = 3.0\text{kV} \\ I_{oh} &= 41.5\text{mA} \;\; ; \;\; V_{in} = 3.5\text{kV} \end{aligned} \tag{22}$$

Table XIII Efficiency of the 3kW, 3kV-to-625V Converter

Vout	Iout	Vin	Iin	Measured η	Eq. (5)
625V	0.27A	2500V	0.12A	56.25%	58.76%
625V	1.16A	2500V	0.34A	85.29%	85.30%
625V	2.06A	2500V	0.57A	90.35%	89.85%
625V	2.94A	2500V	0.80A	91.87%	91.75%
625V	3.83A	2500V	1.04A	92.07%	92.35%
625V	4.69A	2500V	1.27A	92.32%	92.47%
625V	5.55A	2500V	1.5A	92.315%	92.35%

Vout	Iout	Vin	Iin	Measured η	Eq. (5)
625V	0.27A	3000V	0.1	56.25%	55.92%
625V	1.00A	3000V	0.26A	80.13%	81.91%
625V	2.00A	3000V	0.47A	88.65%	88.89%
625V	2.86A	3000V	0.65A	91.67%	90.78%
625V	3.74A	3000V	0.85A	91.67%	91.52%
625V	4.62A	3000V	1.04A	92.55%	91.70%
625V	5.51A	3000V	1.25A	91.83%	91.60%

Vout	Iout	Vin	Iin	Measured η	Eq. (5)
625V	0.27A	3500V	0.09	53.57%	53.55%
625V	1.16A	3500V	0.25A	82.86%	82.42%
625V	2.05A	3500V	0.42A	87.16%	88.01%
625V	2.98A	3500V	0.59A	90.19%	90.00%
625V	3.86A	3500V	0.76A	90.70%	90.64%
625V	4.68A	3500V	0.92A	90.84%	90.76%

The measured efficiency of the converter at each of the input voltages above is shown in **Table XIII**. We can now determine R_{int} for each input voltage according to Eq. (2) as follows:

$$R_{\text{int}}(2.5\text{kV}) = \frac{2.5\text{kV} \times 1.5\text{A} - 625\text{V} \times 5.55\text{A} - 2.5\text{kV} \times 47\text{mA}}{(5.55\text{A})^2} = 5.316\Omega \tag{23}$$

In the same manner, we determine:

$$\begin{aligned} R_{\text{int}}(3\text{kV}) &= 6.055\Omega \\ R_{\text{int}}(3.5\text{kV}) &= 6.967\Omega \end{aligned} \tag{24}$$

These values are used in Eq. (5) and the calculated efficiencies are tabulated in **Table XIII** whence the agreement between Eq. (5) and the measurements are seen to be quite good. A plot of the measured efficiency and Eq. (5) is shown in **Figure 9**.

It is worthwhile to note that the two parameters I_{oh} and R_{int} are not strong functions of the input voltage so that one can either use a single average value for these parameters in P_{oh} and P_{sc} for all input voltages or alternatively use a linearly interpolated value for other input voltages within the allowed input voltage of the converter.

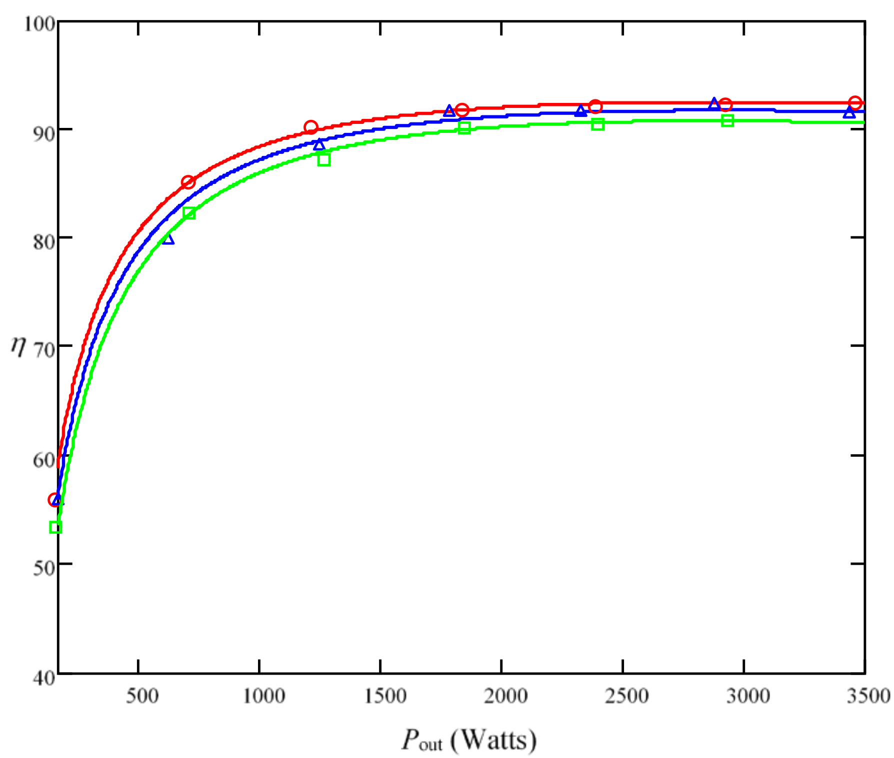

Figure 9 Measured and calculated efficiency curves of the 3kV-to-625V converter.

b) A 28V-to-multiple output dc-to-dc converter

A multiple output converter was built and tested for space application with the following input and output voltages and currents:

Output voltages:

$V_{o1} = 3.3\text{V at } 23\text{A}$

$V_{o2} = 5.1\text{V at } 2\text{A}$

$V_{o3} = 12\text{V at } 400\text{mA}$

Input Voltage

$V_{in} = 22\text{V} - 36\text{V}$

The input power at $V_{in} = 28\text{V}$ with no load on all outputs was measured as follows:

$P_{oh} = 3.495\text{W}$

For each output, P_{sc} is determined by setting the power on the other two outputs to zero. At $V_{in} = 28\text{V}$, the following are measured for each output:

$P_{sc\_1} = 314.47\text{W}$

$P_{sc\_2} = 104\text{W}$

$P_{sc\_3} = 38.26\text{W}$

When these parameters are substituted in the generalized efficiency formula given by Eq. (8), we obtain:

$$\eta(P_o) = \frac{1}{1 + \frac{1}{P_o}\left(3.495 + \frac{P_{o\_1}^2}{314.47} + \frac{P_{o\_2}^2}{104} + \frac{P_{o\_3}^2}{38.26}\right)} \qquad (25)$$

Tables XIV shows the output power of each individual voltage and the total output power and **Table XV** summarizes the measured and computed efficiency given by Eq. (25) and the difference between the two.

Table XIV The output voltages and currents of the three-output converter

V_3.3	V5	V_12	I_3.3	I_5.0	I_12	Po_3.3	Po_5	Po_12	P_out
3.375	5.05	12	0	0	0	0	0	0	0
3.375	5.05	11.5	1.0051	0.206	0.0314	3.392213	1.0403	0.3611	4.793613
3.375	5.05	11.65	2.006	0.406	0.056	6.77025	2.0503	0.6524	9.47295
3.375	5.05	11.73	3.005	0.606	0.081	10.14188	3.0603	0.95013	14.15231
3.375	5.05	11.9	6.006	1.006	0.106	20.27025	5.0803	1.2614	26.61195
3.375	5.05	12.1	9.006	1.506	0.156	30.39525	7.6053	1.8876	39.88815
3.375	5.05	12.3	12.006	2.006	0.206	40.52025	10.1303	2.5338	53.18435
3.375	5.05	12.47	15.007	2.406	0.256	50.64863	12.1503	3.19232	65.99125
3.375	5.05	12.65	18.007	2.807	0.306	60.77363	14.17535	3.8709	78.81988
3.375	5.05	12.66	21.006	2.807	0.306	70.89525	14.17535	3.87396	88.94456
3.375	5.05	12.67	23.007	2.807	0.306	77.64863	14.17535	3.87702	95.701

Table XV Efficiency measurements and comparison with the formula in Eq. (25)

Pout	measured - η	Formular-η	Error
0	0	0	0
4.793613	56.7903008	57.33550438	0.545203576
9.47295	71.0064028	71.87800203	0.871599225
14.15231	76.7950011	78.23446983	1.439468727
26.61195	82.5110735	83.98392445	1.472850954
39.88815	84.271689	85.04443668	0.772747682
53.18435	84.2081472	84.53610129	0.327954087
65.99125	83.3857449	83.43851027	0.052765365
78.81988	82.0118585	82.08781449	0.075955987
88.94456	80.1379953	80.57932025	0.441324951
95.701	78.6829054	79.53364491	0.850739509

A plot of the measured and calculated efficiency points is shown in **Figure 10**.

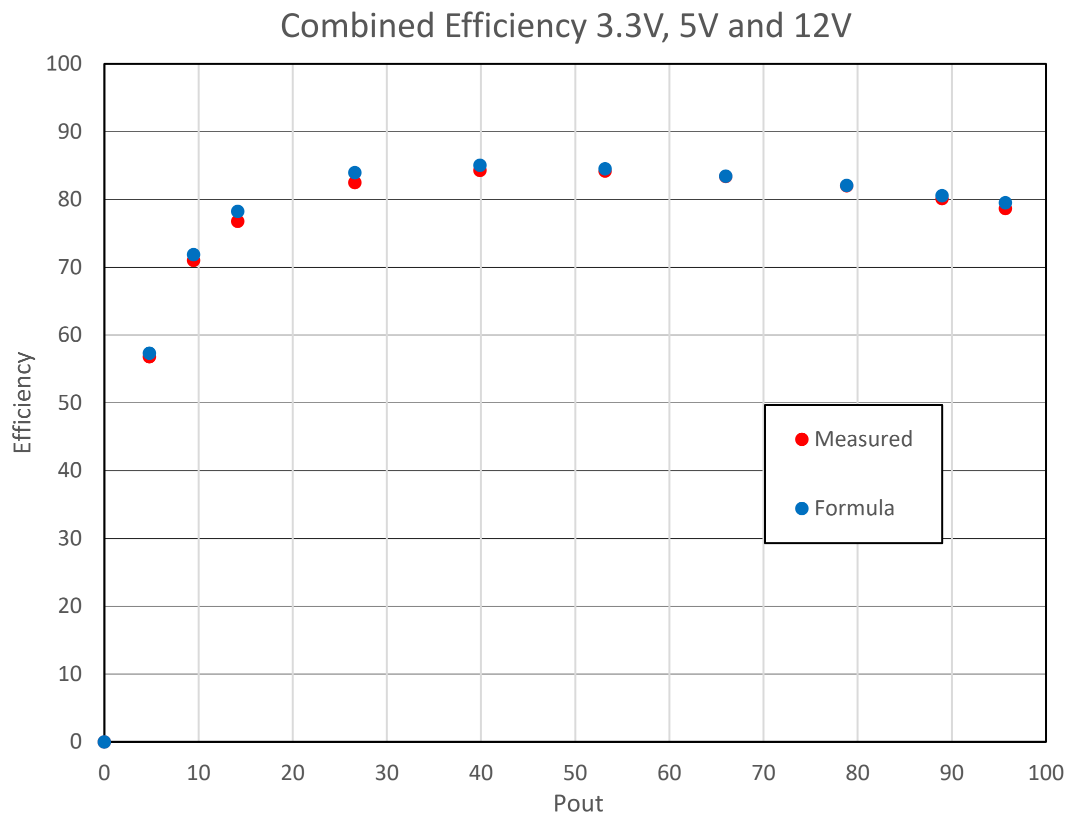

Figure 10 Measured and computed efficiency by Eq. (25) of the multi-output converter at 28V input voltage. See **Table XIV** for the particular individual output powers comprising the total output power.

4. Sensitivity of the efficiency formula to the value of the maximum power.

Whereas one of the two parameters, P_{oh}, is a well defined number and corresponds to the input power at no output load, the second parameter P_{sc} , is related to the maximum rated power of the converter which is not a precise number and certainly can have a range. We are going to show next that, the efficiency function is not terribly sensitive on the value of P_{max}.

In the efficiency equation given by Eq (5), P_{sc}, and hence P_{max}, dominante only at higher power levels where the efficiency is already reasonably high while at very low powers, P_{oh} dominnates role and P_{sc} drops out of the picture. The numerical value of the efficiency equation obtained earlier for the 60W output Vicor converter illustrates this point clearly. When the output power is less than 10W, it is clear that the first two terms in the denominator dominate, while at higher output powers, the first and third terms denominate while the second term drops out. Hence, the value of P_{sc} matters only at higher powers where the efficiency equation can be approximated as follows:

$$\eta = \frac{1}{1 + \frac{P_{oh}}{P_{out}} + \frac{P_{out}}{P_{sc}}} \approx \frac{1}{1 + \frac{P_{out}}{P_{sc}}} \tag{26}$$

Differentiate this with respect to P_{sc} to obtain:

$$d\eta = \eta^2 \frac{P_{out}}{{P_{sc}}^2} dP_{sc} \tag{27}$$

Next, implicitly differentiate Eq. (4) to obtain:

$$P_{sc}\left(dP_{in\_max} - dP_{o\_max}\right) + \left(P_{in\_max} - P_{o\_max} - P_{oh}\right)dP_{sc} = 2P_{o\_max}dP_{o\_max} \tag{28}$$

Solve for dP_{sc} to obtain:

$$dP_{sc} = \frac{2P_{o\_max} - P_{sc}\left(\frac{dP_{in\_max}}{dP_{o\_max}} - 1\right)}{\left(\frac{P_{in\_max}}{P_{o\_max}} - 1 - \frac{P_{oh}}{P_{o\_max}}\right)} \frac{dP_{o\_max}}{P_{o\_max}} \tag{29}$$

We now have:

$$\frac{P_{in\_max}}{P_{o\_max}} = \frac{dP_{in\_max}}{dP_{o\_max}} = \frac{1}{\eta_{p\_max}} \tag{30}$$

$$\frac{P_{oh}}{P_{o\_max}} \ll 1 \tag{31}$$

Using these two approximations, Eq. (29) can now be approximated as:

$$dP_{sc} = \frac{2P_{o\_max} - P_{sc}\left(\frac{1}{\eta_{p\_max}} - 1\right)}{\left(\frac{1}{\eta_{p\_max}} - 1\right)} \frac{dP_{o\_max}}{P_{o\_max}} \tag{32}$$

Recognize that η_{p_max} is *not* the maximum efficiency in Eq. (6) but it is simply the efficiency at P_{o_max}. Hence, according to the first approximation in Eq. (26), we have:

$$\eta_{p\_max} \approx \frac{1}{1 + \frac{P_{o\_max}}{P_{sc}}} \;\Rightarrow\; \frac{1}{\eta_{p\_max}} - 1 \approx \frac{P_{o\_max}}{P_{sc}} \tag{33}$$

Substitution of Eq. (33) in Eq. (32), yields:

$$dP_{sc} = \frac{2P_{o\_max} - P_{o\_max}}{P_{o\_max}{}^2} P_{sc} dP_{o\_max} = \frac{P_{sc}}{P_{o\_max}} dP_{o\_max} \tag{34}$$

Substitution of Eq. (34) in Eq. (27) yields:

$$d\eta = \eta^2 \frac{P_{out}}{P_{sc}{}^2} \frac{P_{sc}}{P_{o\_max}} dP_{o\_max} = \eta^2 \frac{P_{out}}{P_{sc}} \frac{dP_{o\_max}}{P_{o\_max}} \approx \eta^2 \left(\frac{1}{\eta} - 1\right) \frac{dP_{o\_max}}{P_{o\_max}} \tag{35}$$

Divide both sides by the efficiency:

$$\frac{d\eta}{\eta} \approx \eta \left(\frac{1}{\eta} - 1\right) \frac{dP_{o\_max}}{P_{o\_max}} \tag{36}$$

We can see from this last equation that a relative change in the efficiency is related to a relative change in our choice of P_{o_max} by a factor of $\eta\left(\frac{1}{\eta}-1\right)$. For example, if the efficiency of a converter is about 85% at some power below its maximum rated power and we have a 20% range in our assessment of P_{o_max} then the efficiency formula has en error of:

$$d\eta \approx (0.85)^2\left(\frac{1}{0.85}-1\right)0.2 = 0.025 \tag{37}$$

Hence, we see we do not need an exact value of P_{o_max}.

5. Conclusion

The simple efficiency formula, derived from the loss model given in this paper, is intended for use by manufacturers of regulated off-the-shelf dc-to-dc converters to characterize the efficiency of their converters by providing the two parameters, P_{oh} and P_{sc} (or I_{oh} and R_{in}), in their data sheets at different input voltages. The efficiency formulas given in this paper for single and multiple output converters can then be easily evaluated by system designers to fully characterize the efficiency of their overall system which is a particularly useful task in spacecraft power management.

References

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[2] Murata Manufacturing Co. Ltd., “Quarter Brick DC-DC Converter Application Manual MPDKSN00_S Series.”

[3] Murata Manufacturing Co. Ltd., “DC-DC Converter Application Manual MPD6D20*S”

[4] Delta Electronics, Data sheet DS_DNM04SMD10_01222008.